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SOME ALGORITHMIC PROBLEMS IN LINEAR SYSTEMS I.(U)
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SOME ALGORITHMIC PROBLEMS IN LINEAR SYSTEMS I

A. Research Objectives

The purpose of this project, as outlined in the research proposal, is to study algorithmic problems in the following areas of linear systems theory:

(a) Theory of invariant directions of the matrix Riccati equation

(b) The stochastic realization problem See 1473 in back.

(c) Canonical transformations in Hamiltonian systems

(d) Stability theory

especially as these topics relate to a certain class of fast algorithms (non-Riccati algorithms) of Kalman-Bucy filtering and smoothing (replacing the traditional Riccati type equations) and to spectral factorization. Since then, we have extended the research objectives slightly to also include other, but related, aspects of topic (b).

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B. Status of the Research Effort

So far we have studied three of the topics mentioned in the research proposal, namely (i) the stochastic realization problem, (ii) invariant directions and (iii) Hamiltonian systems. These topics are interconnected and there is no sharp line between them. Also work aimed at providing stochastic interpretations of the various smoothing algorithms shattered over the literature is under way and will be reported in next year's technical report.

A list of publications is provided in Section C, and the abstracts of these papers are reproduced in Section D. Ref. 1 gives a comprehensive treatment of the continuous-time stochastic realization problem from both an algorithmic and a probabilistic point of view. The basic problem is to characterize and classify all linear stochastic systems having a given vector process as its output; such a model is called a realization. We distinguish

between internal realizations which are completely determined by the given output process and external realizations which are not. It is shown that the state process of any internal realization can be expressed in terms of two steady-state Kalman filters, one evolving forward in time over the infinite past of the given process and one backward over the infinite future. Connections with the spectral factorization problem are investigated, and a non-Riccati algorithm for generating families of stochastic realizations (or spectral factors), totally ordered with respect to their state covariances, is presented. In Ref. 6, a student of mine, Michele Pavon, modifies some of these results to the discrete-time setting. The basic contribution of this paper, however, is to clarify the connections between the invariant directions of the matrix Riccati equation on one hand and stochastic realization theory and various filtering algorithms on the other hand. The stochastic realization setting turns out to be the right framework for properly understanding the phenomenon of invariant directions, yielding a rather elegant geometric theory. In fact, the invariant directions are invariant over the whole class of stochastic realizations. Ref. 1 and 6 also provide a strict sense backward version of each realization, a result which is most useful in derivations of results involving time-reversal, such as the smoothing problem and the theory of fast (non-Riccati) algorithms.

In References 2,3,4 and 5 we develop a geometric state space theory of stochastic processes, which provides further insight not only into the other topics of this research project but also into such important areas as identification and stochastic and adaptive control. Among other things this theory clarifies the roles of observability and controllability in stochastic systems theory and provides stochastic interpretations for and insight into filtering algorithms. In Ref. 2, given a stationary Gaussian stochastic process y with rational spectral density, we determine all Markovian families of minimal splitting subspaces of H(y), where H(y) is the Hilbert space spanned by the process y; the splitting subspaces act as state spaces. Each such family gives rise to an equivalence class of linear state space models with white noise inputs and y as the output, systems of the type used in modeling engineering problems. In fact, we obtain all such models which can be constructed in terms of the given process (internal realizations). In Ref. 3 we develop a more general theoretical framework which will allow us to obtain a similar theory for non-stationary processes and to obtain also the models which cannot be constructed in terms of y (external realizations). In Ref. 4 the powerful theory of Hardy spaces is applied to the problem, yeilding explicit solutions also for infinite-dimensional systems; this will enable us to apply the theory to time-delay systems and distributed parameter systems. Finally, in Ref. 5, this Hardy space theory is generalized to the multivariate case. A new approach to the basic geometric problem is presented, more suitable for the multivariate setting. It is shown that each state space is the isomorphic image of the closure of a certain Hankel operator, which facilitates the application of the infinite-dimensional deterministic realization theory of Brockett, Baras and Fuhrmann to stochastic systems.

Faris Badawi is writing a Ph.D.-thesis under my direction, which considers the above mentioned modelling problems from a more deterministic and algorithmical point of view. Among other things he has developed a discrete version of the non-Riccati algorithm for generating spectral factors, which can be used both in the discrete-time and continuous-time stochastic realization problem. (The singular realization problem has also been studied.) He is also developing a comprehensive framework for discrete non-Riccati algorithms of filtering and realization in the Hamiltonian setting described in the research proposal. This is not so hard in continuous time, but the discrete-time case offers some interesting complications. The goal is to develop physical interpretations of these factorization results, which will aid us in understanding the related questions of stability, etc. Except for the stationary case, the present factorization results in discrete time are rather ad hoc, giving little insight into its physical meaning. Some preliminary steps have been taken to relate this to algorithms for the algebraic Riccati equation. These matters will be futher expanded upon in next year's technical report.

C. Publications

1. A. Lindquist and G. Picci, On the stochastic realization problem, SIAM J. Control and Optimization 17 (May 1979); to appear (partly supported by AFOSR and partly by NSF).

2. A. Lindquist and G. Picci, A state space theory for stationary stochastic processes, Proc. 21st Midwest Symposium on Circuits and Systems, August 1978, 108-113 (invited paper).

3. A. Lindquist, G. Picci and G. Ruckebusch, On minimal splitting subspaces and Markovian representations, Mathematical Systems Theory 12 (May 1979); to appear.

4. A. Lindquist and G. Picci, A Hardy space approach to the stochastic realization problem, Proc. 1978 Conf. Decision and Control, San Diego, 933-939 (invited paper).

5. A. Lindquist and G. Picci, Realization theory for multivariate stationary Gaussian processes I: State space construction, Proc. 4th Intern. Symp. Math. Theory of Networks and Systems, July 1979, Delft, Holland (invited paper).

6. M. Pavon, Stochastic realization and invariant directions of the matrix Riccati equation, SIAM J. Control and Optimization 17 (1979); to appear (partly supported by AFOSR and partly by NSF).

D. Abstracts of Publications

1. "On the stochastic realization problem":

Given a mean-square continuous stochastic vector process y with stationary increments and a rational spectral density Φ such that $\Phi(\infty)$ is finite and nonsingular, consider the problem of finding all minimal Gauss-Markov representations (stochastic realizations) of y. All such realizations are characterized and classified with respect to deterministic as well as probabilistic properties. It is shown that only certain realizations (internal stochastic realizations) can be determined from the given output process y. All others (external stochastic realizations) require that the probability space be extended with an exogeneous random component. A complete characterization of the sets of internal and external stochastic realizations is provided. It is shown that the state process of any internal stochastic realization can be expressed in terms of two steady-state Kalman-Bucy filters, one evolving forward in time over the infinite past and one backward over the infinite future. An algorithm is presented which generates families of external realizations defined on the same probability space and totally ordered with respect to state covariances.

2. "A state space theory for stationary stochastic processes":

Consider a stationary Gaussian stochastic process $\{y(t);t \in R\}$ with a rational spectral density, and let H(y) be the Hilbert space spanned by it. The problem of determining all stationary and purely nondeterministic families of minimal splitting subspaces of H(y) is considered; the splitting subspaces constitute state-space for the process y. It is shown that some of these families are Markovian, and they lead to internal stochastic realizations. A complete characterization of all Markovian and non-Markovian families of minimal splitting subspaces is provided. Many of the basic results hold without the assumption of rational spectral density.

3. "On minimal splitting subspaces and Markovian representations":

Given a Hilbert space H, let H_1 and H_2 be two artibrary subspaces.

The problem of finding all minimal splitting subspaces of H with respect to H_1 and H_2 is solved. This result is applied to the stochastic realization problem. Each minimal stochastic realization of a given vector process y defines a family of state spaces. It is shown that these families are precisely those families of minimal splitting subspaces (with respect to the past and the future of y) which satisfy a certain growth condition.

4. "A Hardy space approach to the stochastic realization problem":

Given a purely nondeterministic mean-square continuous Gaussian stationary stochastic process we consider the problem of characterizing all minimal splitting subspaces X which evolve in time in a Markovian fashion. Let $H^{+/-}$ and $H^{-/+}$ be the projection of the future of the given process onto the past and the past onto the future respectively. It is shown that the family {X} of minimal Markovian splitting subspaces can be isomorphically described as a partially ordered family of subspaces of the form $X \cong j\mathfrak{X}^*$ where $\mathfrak{X}^* \cong H^{-/+}$ and j ranges over the family of all inner divisors of a fixed inner function j_* uniquely defined by $H^{+/-}$. The procedure is illustrated with an application to a process with a rational spectral density.

5. "Realization theory for multivariate stationary Gaussian processes I: State Space construction".

A new approach to the problem of determining all internal minimal Markovian splitting subspaces of a given stationary process y is presented which is particularly suitable for solving the multivariate stochastic realization problem. Under the assumption that y is strictly noncyclic it is shown that each such state space can be expressed in terms of two subspaces spanned by the pasts of a corresponding pair of Wiener processes, one being a forward and the other a backward innovation process for the state space. This makes it possible to solve the problem in the Hardy space H_2^+, and each state space is seen to be the isomorphic image of the closure of a certain Hankel operator in H_2^+. This result will facilitate the application of existing infinite-dimensional deterministic realization theory to obtain explicit Markovian representations; this is the topic of Part II.

6. "Stochastic realization and invariant directions of the matrix Riccati equation":

Invariant directions of the Riccati difference equation of Kalman filtering are shown to occur in a large class of prediction problems and to be related to a certain invariant subspace of the transpose of the feedback matrix. The discrete time stochastic realization problem is studied in its deterministic as well as probabilistic aspects. In particular a new derivation of the classification of the minimal markovian representations of the given process z is presented which is based on a certain backward filter of the innovations. For each markovian representation which can be determined from z the space of invariant directions is decomposed into two subspaces, one on which it is possible to predict the state process without error forward in time and one on which this can be done backward in time.

E. Professional Personnel Associated with the Research Effort

Anders Lindquist (principal investigator), Assoc. Professor

Michele Pavon, graduate student, awarded Ph.D. degree May 1979 on a thesis entitled "Duality Theory, Stochastic Realization and Invariant Directions for Linear Discrete-Time Stochastic Systems" (dissertation director: A. Lindquist); the first 3/4 of this thesis is identical to reference 6 reported under C.

Faris Badawi, graduate student, estimated to obtain his Ph.D. in May 1980.

Giorgio Picci (consultant), Professor, University of Padova, Italy.

Guy Ruckebusch (consultant), Ph.D., Ecole Polytechnique, Palaiseau, France.

F. Interactions

Results obtained within this project have been (or will be) presented on the following occations:

Professional Meetings:

1. Workshop on Current Topics in Communications, Washington University, St. Louis, March 6-7, 1978 (A. Lindquist, invited speaker).

2. Workshop on Fast and Square-Root Algorithms, Universite Catholique de Louvain, Belgium, June 1978 (A. Lindquist, invited speaker).

3. 21st Midwest Symposium on Circuits and Systems, Ames,Iowa, August 14-15, 1978 (A. Lindquist, invited speaker).

4. 17th Conference on Decision and Control, San Diego, January 10-12, 1979 (A. Lindquist; SIAM-paper, invited).

5. Special Session on Integral Equations with Emphasis on Fredholm and Hammerstein Equations, 85th annual meeting of the AMS, Biloxi, January 24-28, 1979 (A. Lindquist, invited speaker; could not attend due to illness).

6. IEEE International Symposium on Information Theory, Grignano, Italy, June 25-29, 1979 (A. Lindquist or G. Picci).

7. 4th International Symposium on Mathematical Theory of Networks and Systems, Delft, Holland, July 3-6, 1979. (A. Lindquist and G. Picci, invited speakers).

8. Special Session of Stochastic Realization Theory (organized by A. Lindquist), 1979 International Conference on Information Sciences and Systems, Patras, Greece, July 9-13, 1979 (G. Picci).

Seminars and Colloquia:

1. Washington University, November 1978 (G. Picci)

2. M.I.T., November 1978 (G. Picci)

3. M.I.T., November 1978 (M. Pavon)

4. Toronto, April 9, 1979 (A. Lindquist)

5. Brown University, April 10-11, 1979 (A. Lindquist; invited by H. Kushner)

6. M.I.T., April 12, 1979 (A. Lindquist)

7. University of Maryland, tentatively May 1979 (A. Lindquist)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A comprehensive study of the stochastic realization problem was undertaken both in its algorithmic and structural aspects. A geometric state space theory of linear stochastic systems was obtained, providing a complete characterization of all stochastic realizations. The phenomenon of invariant directions of the matrix Riccati equation was investigated in this framework, clarifying its connections to filtering algorithms as well as to geometric aspects of linear systems theory. Non-Riccati algorithms for generating spectral factors were introduced. Work was initiated to

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20. Abstract (continued)

cont.
develop a comprehensive framework for algorithms of this type through transformations in Hamiltonian systems.